

# Maintenance of Data Cubes and Summary Tables in a Warehouse

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## Abstract

Data warehouses contain large amounts of information, often collected from a variety of independent sources. Decision-support functions in a warehouse, such as *on-line analytical processing* (OLAP), involve hundreds of complex aggregate queries over large volumes of data. It is not feasible to compute these queries by scanning the data sets each time. Warehouse applications therefore build a large number of *summary tables*, or materialized aggregate views, to help them increase the system performance.

As changes, most notably new transactional data, are collected at the data sources, all summary tables at the warehouse that depend upon this data need to be updated. Usually, source changes are loaded into the warehouse at regular intervals, usually once a day, in a batch window, and the warehouse is made unavailable for querying while it is updated. Since the number of summary tables that need to be maintained is often large, a critical issue for data warehousing is how to maintain the summary tables efficiently.

In this paper we propose a method of maintaining aggregate views (the *summary-delta table* method), and use it to solve two problems in maintaining summary tables in a warehouse: (1) how to efficiently maintain a summary table while minimizing the batch window needed for maintenance, and (2) how to maintain a large set of summary tables over the same base tables. We show that much of the work required for maintaining one summary table by the summary-delta method can be re-used in maintaining other summary tables, so that a set of summary tables can be maintained efficiently.

While several papers have addressed the issues relating to choosing and materializing a set of summary tables, this is the first paper to address maintaining summary tables efficiently.

## 1 Introduction

Data warehouses contain information that is collected from multiple, independent data sources and integrated into a common repository for querying and analysis. Often, data warehouses are designed for *on-line analytical processing* (OLAP), where the queries aggregate large volumes of data in order to detect trends and anomalies. In order to speed up query processing in such environments, warehouses usually contain a large number of *summary tables*, which represent materialized aggregate views of the base data collected from the sources. The summary tables group the base data along various *dimensions*, corresponding to different sets of group-by attributes, and compute various aggregate functions, often called *measures*. As an example, the cube operator [GBLP96] can be used to define several such summary tables with one statement.

As changes are made to the data sources, the warehouse views must be updated to reflect the changed state of the data sources. The views either can be recomputed from scratch, or incremental maintenance techniques [BC79, SI84, RK86, BLT86, Han87, SP89, QW91, Qua96, CW91, GMS93, GL95, LMSS95, ZGHW95] can be used to calculate the changes to the views due to the source changes. Using the incremental maintenance approach, the warehouse can be updated either *immediately* as soon as a change from a source is

received, or the update can be *deferred* until a time when a large batch of updates is applied to the warehouse at once.

Most of the work and implementation on view maintenance has involved the immediate case [BLT86, QW91, CW91, GL95]. Immediate maintenance has the downside that each update into the warehouse incurs the overhead of updating the views. This overhead increases with the number of views and their complexity. Another problem with immediate maintenance in a warehousing environment is that data warehouse applications often require that the state of the views not change while the warehouse is being accessed, so that readers see a consistent snapshot of the warehouse across a sequence of multiple queries during analysis [AL80]. For these reasons, warehouses are typically maintained in deferred mode, with the source changes received during the day applied to the views in a nightly batch window, during which time the warehouse is unavailable to readers.

The nightly batch window involves updating the base tables (if any) stored at the warehouse, and maintaining all the materialized summary tables. The problem with this approach is that the warehouse is typically unavailable to readers while the views are being maintained, due to the large number of updates that need to be applied. Since the warehouse must be made available to readers again by the next morning, the time required for maintenance is often a limiting factor in the number of summary tables that can be made available in the warehouse. Because the number of summary tables available has such a significant impact on OLAP query performance, maintaining the summary tables efficiently is crucial.

This paper addresses the issue of efficiently maintaining a set of summary tables in a data warehouse. Using efficient incremental maintenance techniques, it is possible to increase the number of summary tables available in the warehouse, or alternatively, to decrease the time that the warehouse is unavailable to readers. The paper includes the following contributions:

- Incremental maintenance techniques for summary tables are given. Except for [Qua96], previous work on view maintenance has touched upon aggregate views only briefly [GMS93, GL95]. We propose a new paradigm, called the summary-delta tables method, for maintenance of aggregate views.
- A general strategy to minimize the batch time needed for maintenance is to split the maintenance work into *propagate* and *refresh* functions. Propagate can occur outside the batch window, while refresh occurs inside the batch window. The propagate and refresh split for relational algebra was originally presented and formalized in [CGL+96]. We use the propagate and refresh approach of [CGL+96], and extend it to aggregate views by giving algorithms that split the maintenance work required for summary tables into propagate and refresh functions.
- The work required to maintain one summary table often can be re-used toward maintaining other summary tables. Thus, a set of summary tables can be maintained more efficiently together than maintaining each summary table in isolation. We show how multiple summary tables can be related so that their maintenance can take advantage of the computation done to maintain other tables.

**Paper outline:** Section 2 presents a motivating example illustrating the importance of efficient incremental maintenance of summary tables. Background and notation is given in Section 3. Section 4 presents propagate and refresh functions for maintaining individual summary tables. Section 5 explains how multiple summary tables can be maintained efficiently together. The summary-delta table method described in this paper has been implemented, and a performance study based upon it is shown in Section 6. Related work and conclusions appear in Section 7.

## 2 Motivating Example

Consider a warehouse of retail information, with point-of-sale (**pos**) data from hundreds of stores. The point of sale data is stored in the warehouse in a large **pos** table, called a *fact table*, that contains a tuple for each

item sold in a sales transaction. Each tuple has the format

```
pos(storeID, itemID, date, qty, price).
```

The attributes of the tuple are the id of the store selling the item, the id of the item sold, the date of the sale, the quantity of the item sold, and the selling price of the item. The `pos` table is allowed to contain duplicates, for example, when an item is sold in different transactions in the same store on the same date.

In addition, a warehouse will often store *dimension tables*, which contain information related to the fact table. Let the `stores` and `items` tables contain store information and item information, respectively. The key of `stores` is `storeID`, and the key of `items` is `itemID`.

```
stores(storeID, city, region).
items(itemID, name, category, cost).
```

Data in dimension tables often represents *dimension hierarchies*. A dimension hierarchy is essentially a set of functional dependencies among the attributes of the dimension table. For our example we will assume that in the stores dimension hierarchy, `storeID` functionally determines `city`, and `city` functionally determines `region`. In the items dimension hierarchy, `itemID` functionally determines `name`, `category`, and `cost`.

In order to answer aggregate queries quickly, a warehouse will often store a number of summary tables, which are materialized views that aggregate the data in the fact table, possibly after joining it with one or more dimension tables. Figure 1 shows four summary tables, each defined as a materialized SQL view. We assume that these views have been chosen to be materialized, either by the database administrator, or by using an algorithm such as [HRU96].

```
CREATE VIEW SID_sales(storeID, itemID, date, TotalCount, TotalQuantity) AS
SELECT storeID, itemID, date, COUNT(*) AS TotalCount, SUM(qty) AS TotalQuantity
FROM pos
GROUP BY storeID, itemID, date

CREATE VIEW sCD_sales(city, date, TotalCount, TotalQuantity) AS
SELECT city, date, COUNT(*) AS TotalCount, SUM(qty) AS TotalQuantity
FROM pos, stores
WHERE pos.storeID = stores.storeID
GROUP BY city, date

CREATE VIEW SiC_sales(storeID, category, TotalCount, EarliestSale, TotalQuantity) AS
SELECT storeID, category, COUNT(*) AS TotalCount, MIN(date) AS EarliestSale, SUM(qty) AS TotalQuantity
FROM pos, items
WHERE pos.itemID = items.itemID
GROUP BY storeID, category

CREATE VIEW sR_sales(region, TotalCount, TotalQuantity) AS
SELECT region, COUNT(*) AS TotalCount, SUM(qty) AS TotalQuantity
FROM pos, stores
WHERE pos.storeID = stores.storeID
GROUP BY region
```

Figure 1: Example summary tables

Note that the names of the views have been chosen to reflect the group-by attributes. The character S represents storeID, I represents itemID, and D represents date. The notation sC represents the city for a store, sR represents the region for a store, and iC represents the category for an item. For example, the name SiC\_sales implies that storeID and category are the group-by attributes in the view definition.

The views of Figure 1 could represent four of the possible points on a “data cube” as described in [GBLP96], except for the use of date as both a dimension and a measure. Another difference between this paper and previous work on data cubes is that in previous work the data being aggregated comes solely from the fact table, with dimension hierarchy information obtained implicitly. As mentioned earlier, data warehouses typically store dimension hierarchy information explicitly in dimension tables; in this paper we extend the data-cube concept to include explicit joins with dimension tables (see Section 3.3).

As sales are made, changes representing the new point-of-sale data come into the warehouse. For the reasons mentioned in Section 1, most warehouses do not apply the changes immediately. Instead, changes are deferred and applied to the base tables and summary tables in the warehouse at night in a single batch. Recall that deferring the changes allows analysts that query the warehouse to see a consistent snapshot of the data throughout the day, and can make the maintenance more efficient.

Although it is often the case that changes to a warehouse involve only insertions, for the sake of example in this paper we will assume that the changes involve both insertions and deletions. In order to correctly maintain an aggregate view in the presence of deletions it is necessary to include a COUNT(\*) aggregate function in the view. Having COUNT(\*) makes it possible to determine when all tuples in a group have been deleted (*i.e.*, when COUNT(\*) for the group becomes 0), implying the deletion of the tuple for the group in the view. We have included COUNT(\*) explicitly in the example views above, but it also could be added implicitly when the view is materialized in the warehouse.

For simplicity of presentation, we will usually assume in this paper that maintenance is performed in response to changes only to the fact table, and that the columns being aggregated do not include null values. However, the algorithms we present are easily extended to handle changes also to the dimension tables, as well as nulls in the aggregated columns. The effect of changes to dimension tables is considered in Section 4.1.4, and the effect of nulls in the aggregated columns is considered in Section 3.1.

## 2.1 Maintaining a single summary table

We will this section we illustrate by example our summary-delta table method, using it to maintain the SID\_sales summary table of Figure 1. Later in Section 2.2, we show that much of the work in maintaining SID\_sales can be re-used to maintain the other summary tables in the figure. The complete algorithms for maintaining a single summary table and a set of summary tables appear in Sections 4 and 5 respectively.

An important aspect of our maintenance algorithm is that the maintenance process is divided into two functions: propagate and refresh. The work of computing a summary-delta table happens within the propagate function, which can take place without locking the summary tables so that the warehouse can continue to be made available for querying by clients. Summary tables are not locked until the refresh function, during which time the summary table is updated from the summary-delta table.

**Propagate:** The propagate function involves creating a *summary-delta table* from the deferred set of changes. The summary-delta table represents the net changes to the summary table due to the changes to the fact table. Let the deferred set of insertions be stored in table pos\_ins and the deferred set of deletions be stored in table pos\_del. Then the summary-delta table is derived using the following SQL statement, without accessing the base pos table.

```
CREATE VIEW sd_SID_sales (storeID, itemID, date, sd_Count, sd_Quantity) AS
SELECT storeID, itemID, date, SUM(_count) AS sd_Count, SUM(_quantity) AS sd_Quantity
FROM ( (SELECT storeID, itemID, date, 1 as _count, qty as _quantity FROM pos_ins)
UNION ALL
```

```

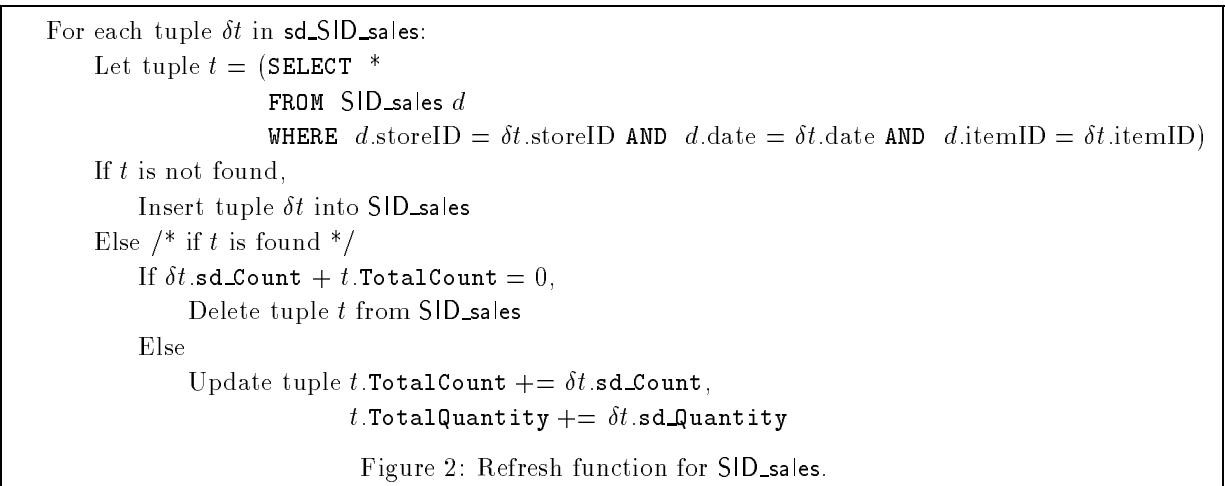
        (SELECT storeID, itemID, date, -1 as _count, -qty as _quantity FROM pos_del)
    )
GROUP BY storeID, itemID, date

```

To compute the summary-delta table, we first perform a projection on the inserted and deleted tuples so that we have 1 for count and qty for quantity from the inserted tuples, and the negative of those values from the deleted tuples. We then take the union of this result and aggregate it, grouping by the same group-by attributes as in the summary table. The resulting aggregate function values represent the net changes to the corresponding aggregate function values in the summary table. The propagate function is explained fully in Section 4.1.

**Refresh:** The refresh function applies the net changes represented in the summary-delta table to the summary table. The function to refresh `SID_sales` appears in Figure 2, and is described below. It takes as input the summary-delta table `sd_SID_sales`, and the summary table `SID_sales`, and updates the summary table to reflect the changes in the summary-delta table. For simplicity, we assume here that there are no null values in `pos`. Null values will be considered later when deriving the generic refresh algorithm.

The refresh function has been designed to run as quickly as possible. Except for certain cases involving `MIN` and `MAX` (see Section 4.2), the refresh function does not require access to the base `pos` table, and all aggregation is performed in the propagate function. Each tuple in the summary-delta table causes a single update to the summary table, and each tuple in the summary table is updated at most once.



Intuitively, the refresh function of Figure 2 can be written as an embedded SQL program using cursors as follows. A cursor  $c_1$  is opened to iterate over each tuple  $\delta t$  in the summary-delta table `sd_SID_sales`. For each  $\delta t$ , a query is issued and a second cursor  $c_2$  is opened to find a matching tuple  $t$  in the summary table `SID_sales` (there is at most one matching  $t$  since the match is on the group-by attributes). If a matching tuple  $t$  is not found, then the  $\delta t$  tuple is inserted into the summary table. Otherwise, if  $t$  is found it is updated or deleted using cursor  $c_2$ , depending upon whether all tuples in  $t$ 's group have been deleted. The refresh function is explained fully in Section 4.2, including an explanation of how it can be optimized when certain integrity constraints on the changes hold.

## 2.2 Maintaining multiple summary tables

We now give propagate functions that create summary deltas for the remaining summary tables of Figure 1. Efficiently maintaining multiple summary tables together allows more opportunity for optimization than main-

taining each summary table individually, because the summary-delta table computed for the maintenance of one summary table often can be used to compute summary-delta tables for other summary tables. Since a summary-delta table already involves some aggregation over the changes to the base tables, it is likely to be smaller than the changes themselves, so using a summary-delta table to compute other summary-delta tables will likely require fewer tuple accesses than computing each summary-delta table from the changes directly.

The queries defining summary-delta tables for `sCD_sales`, `SiC_sales`, and `sR_sales` are shown in Figure 3. The summary-delta tables for `sCD_sales` and `SiC_sales` both reference the summary-delta table for `SID_sales`, and the summary-delta table for `sR_sales` references the summary-delta table for `sCD_sales`.

```
CREATE VIEW sd_sCD_sales(city, region, date, sd_Count, sd_Quantity) AS
SELECT city, region, date, sum(sd_Count) AS sd_Count, sum(sd_Quantity) AS sd_Quantity
FROM sd_SID_sales, stores
WHERE sd_SID_sales.storeID = stores.storeID
GROUP BY city, region, date

CREATE VIEW sd_SiC_sales(storeID, category, sd_Count, sd_EarliestSale, sd_Quantity) AS
SELECT storeID, category, sum(sd_Count) AS sd_Count, min(date) AS sd_EarliestSale,
       sum(sd_Quantity) AS sd_Quantity
FROM sd_SID_sales, items
WHERE sd_SID_sales.itemID = items.itemID
GROUP BY storeID, category

CREATE VIEW sd_sR_sales(region, sd_Count, sd_Quantity) AS
SELECT region, sum(sd_Count) AS sd_Count, sum(sd_Quantity) AS sd_Quantity
FROM sd_sCD_sales
GROUP BY region
```

Figure 3: Propagate Functions for `sCD_sales`, `SiC_sales`, and `sR_sales`.

Notice that the summary-delta table `sd_sCD_sales` includes the region attribute, which is not necessary to maintain `sCD_sales`. Region is included so that later in the definition of `sd_sR_sales` we do not need to join `sd_sCD_sales` with `stores`. Including region in `sd_sCD_sales` does not affect the maintenance of `sCD_sales` because in the dimension hierarchy for cities we have specified that city functionally determines region, *i.e.*, every city belongs to a single region, so grouping by (city, region, date) results in the same groups as grouping by (city, date).

The refresh functions corresponding to the above summary-delta tables are not given in this section. In general they follow in a straightforward fashion from the example refresh function for `SID_sales` given in Section 2.1, with the exception of the `MIN` aggregate function in `SiC_sales`. In Section 4.2 we show how the refresh function handles `MIN` and `MAX` aggregate functions.

### 2.3 Previous Aggregation Techniques

The technique of [GMS93] works by computing a set of insertions and deletions (combined into one delta set  $\Delta$  with positive and negative counts) for each materialized view. For aggregate views, whenever a tuple is inserted or deleted into a group in the base table, the tuple for the corresponding group is deleted from the materialized view. All such tuples are then recomputed from the base tables.

In [GL95], maintenance expressions for views having duplicate semantics are given, including an example showing how to update an aggregate function (without group by) using the values of the aggregate function applied to the set of insertions and deletions to the base data. However, maintenance of aggregate views

with group-by attributes is not considered. [JMS95] discusses the computational complexity of immediately maintaining a single aggregate view in response to a single insertion into a *chronicle* (sequence of tuples).

The maintenance algorithms of [Qua96] extend the maintenance expressions of [GL95] by including expressions for updating aggregate views with group-by attributes. The major differences between the current paper and that of [Qua96] are: [Qua96] gives general maintenance expressions for maintaining views with aggregation, with little attention to efficiency, while the current paper develops a method for maintaining a restricted class of aggregate views, called the generalized cube views, which commonly appears as summary tables in a data warehouse, and presents a method to maintain views of this type very efficiently. The maintenance algorithm of [Qua96] focuses mainly on deleting and inserting tuples in the view, while the summary-delta table method updates the view tuples. In addition, [Qua96] considers maintaining only one aggregate view at a time, while this work shows how to reduce the total amount of work by maintaining multiple summary tables together. Also note that [Qua96] was presented at a workshop without a formal proceedings.

## 2.4 Importance of efficient incremental maintenance

We will illustrate the benefit that can be obtained by maintaining summary tables using the summary-delta algorithm presented in this paper. For the summary tables of Figure 1, we contrast the cost of recomputing from scratch against the cost of incremental maintenance using the counting algorithm of [GMS93] and the summary-delta algorithm proposed in this paper, and further contrast the cost of maintaining each summary table in isolation against the cost when summary-delta tables are re-used to maintain other summary-delta tables.

Summary table	Tuple Reads and Writes			
	Recomputation	Summary-delta maintenance		Counting[GMS93] maintenance
		Individual	Reuse deltas	
SID_sales	1,100,000	11,000	11,000	22,000
sCD_sales	110,000	10,100	1,100	20,200
SiC_sales	102,000	11,000	2,000	512,000
sR_sales	2,010	10,010	110	1,010,020
Total cost	1,314,010	42,110	14,210	1,564,220

Table 1: Benefits of summary-delta Maintenance Algorithm.

Given a particular database (described in Appendix A), Table 1 shows the number of tuple reads and writes required to recompute and incrementally maintain each of the summary tables. The first column in Table 1 shows the number of tuple accesses required to recompute each of the four summary tables, assuming that summary tables are recomputed from other summary tables when possible, so that once the SID\_sales summary table has been computed it can be used to compute the sCD\_sales and SiC\_sales summary tables, and so on. The next two columns show the number of tuple accesses required to maintain each of the four summary tables using the summary-delta table method. The second column assumes that each summary table is maintained directly from the changes to the pos table, while the third column assumes that summary-delta tables created for one summary table are re-used in the creation of summary-delta tables for other summary tables (using the queries of Figure 3). The fourth column uses the counting algorithm [GMS93] to do incremental maintenance. A discussion of how the numbers in Table 1 are derived appears in Appendix A.

Table 1 shows that the summary-delta algorithm leads to significant savings over recomputation and the counting algorithm. Further, the summary-delta algorithm benefits a lot by re-using summary-delta tables. We also see that the counting algorithm can be even worse than recomputation for relatively small change sets. These conclusions derived from a very simple analytical model here are borne out by our implementation

and performance study (Section 6).

### 3 Background and Notation

In this section we review the concepts of self-maintainable aggregate functions (Section 3.1), data cube (Section 3.2), and the computation lattice corresponding to a data cube (Section 3.3).

#### 3.1 Self-maintainable aggregate functions

In Gray *et al.* [GBLP96], aggregate functions are divided three classes: *distributive*, *algebraic*, and *holistic*. Distributive aggregate functions can be computed by partitioning their input into disjoint sets, aggregating each set individually, then further aggregating the (partial) results from each set into the final result. Amongst the aggregate functions found in standard SQL, `COUNT`, `SUM`, `MIN`, and `MAX` are distributive. For example, `COUNT` can be computed by summing partial counts. Note however, if the `DISTINCT` keyword is used, as in `COUNT(DISTINCT E)` (count the distinct values of  $E$ ) then these functions are no longer distributive.

Algebraic aggregate functions can be expressed as a scalar function of distributive aggregate functions. Average is algebraic, since it can be expressed as `SUM/COUNT`. From now on we will assume that if a view is supposed to contain the `AVG` aggregate function, the materialized view will contain instead the `SUM` and `COUNT` functions.

Holistic aggregate functions cannot be computed by dividing into parts. Median is an example of a holistic aggregate function. We will not consider holistic functions in this paper.

**Definition 3.1 (Self-maintainable aggregate functions):** A set of aggregate functions is *self-maintainable* if the new value of the functions can be computed solely from the old values of the aggregation functions and from the changes to the base data. Aggregate functions can be self-maintainable with respect to insertions, with respect to deletions, or both. ■

In order for an aggregate function to be self-maintainable it must be distributive. In fact, all distributive aggregate functions are self-maintainable with respect to insertions. However, not all distributive aggregate functions are self-maintainable with respect to deletions. The `COUNT(*)` function can help to make certain aggregate functions self-maintainable with respect to deletions, by helping to determine when all tuples in the group (or in the full table if a group-by is not performed) have been deleted, so that the grouped tuple can be deleted from the view.

The function `COUNT(*)` is always self-maintainable with respect to deletions. Including `COUNT(*)` also makes the function `COUNT(E)`, (count the number of non-null values of  $E$ ), self-maintainable with respect to deletions. If nulls are not allowed in the input, then `COUNT(*)` also makes `SUM(E)` self-maintainable with respect to deletions. In the presence of nulls, both `COUNT(*)` and `COUNT(E)` are required to make `SUM(E)` self-maintainable, as explained below:

**Nulls and the SUM aggregation function:** Nulls are ignored when computing aggregate values in SQL-92 [MS93]. Therefore, if a non-empty relation  $R$  contains only tuples having null values for an attribute  $R.A$ , the query

```
SELECT SUM(R.A)
FROM R
```

would return a single tuple with a null value (*The query does not return 0 as the answer*).

The semantics for nulls implies that for `SUM(E)`, the sum of an expression  $E$ , if a tuple contributing the last non-null value of  $E$  is deleted (`COUNT(E) = 0`) and there are still tuples in the group (`COUNT(*) > 0`), the value of `sum(E)` must be set to null. Thus, in the presence of nulls both `COUNT(*)` and `COUNT(E)` are required to maintain `SUM(E)`.



**MIN and MAX functions:** **MIN** and **MAX** are not self-maintainable with respect to deletions, and cannot be made self-maintainable. For instance, when a tuple having the minimum (maximum) value is deleted, the new minimum (maximum) value for the group must be recomputed from the changes and the base data. Including **COUNT(\*)** can help a little (if **COUNT(\*)** reaches 0, there is no other tuple in the group, so the group can be deleted), but **COUNT(\*)** cannot make **MIN** and **MAX** self-maintainable. (If **COUNT(\*)** > 0 after a tuple having minimum (maximum) value is deleted, we still need to look up the base table.) **COUNT(E)** can also help in maintaining **MIN(E)** and **MAX(E)** (If **Count(\*)** > 0 and **COUNT(E)** = 0, then **MIN(E)**= null), but **COUNT(E)** also cannot make **MIN** and **MAX** self-maintainable (if **Count(\*)** > 0 and **COUNT(E)** > 0, and a tuple having minimum (maximum) value is deleted, then we need to look up the base table).

### 3.2 Data Cube

The data cube [GBLP96] is a convenient way of thinking about multiple aggregate views, all derived from a fact table using different sets of group-by attributes. Data cubes are popular in OLAP because they provide an intuitive way for data analysts to navigate various levels of summary information in the database. In a data cube, attributes are categorized into *dimension attributes*, on which grouping may be performed, and *measures*, which are the results of aggregate functions.

**Cube Views:** A data cube with  $k$  dimension attributes is a shorthand for  $2^k$  cube views, each one defined by a single **SELECT -FROM -WHERE -GROUP BY** block, having identical aggregation functions, identical **FROM** and **WHERE** clauses, no **HAVING** clause, and one of the  $2^k$  subsets of the dimension attributes as the groupby columns.

**EXAMPLE 3.1** An example data cube for the **pos** table of Section 2 is shown in Figure 4(a) as a lattice structure. (Ignore Figure 4(b)) for now.) Construction of the lattice corresponding to a data cube was first

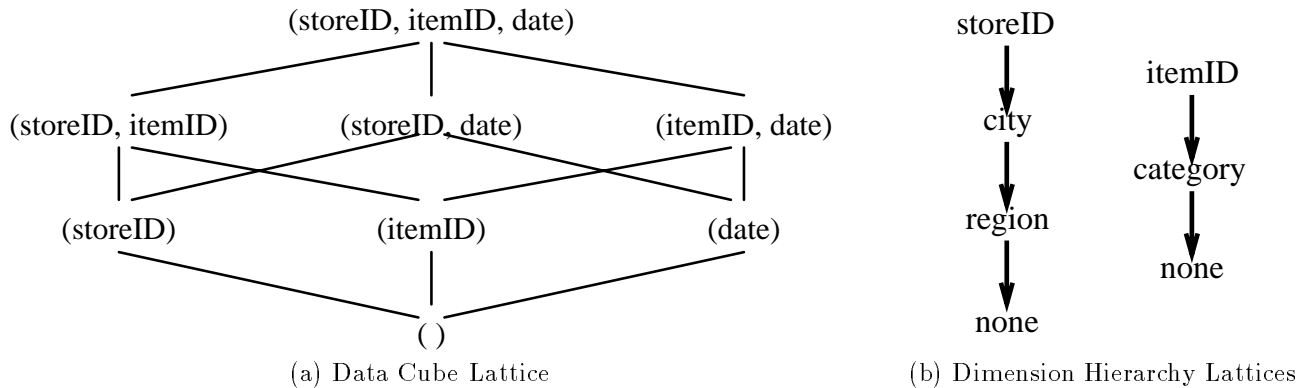


Figure 4: Example Lattices.

introduced in [HRU96]. The dimension attributes of the data cube are **storeID**, **itemID**, and **date**, and the measures are **COUNT(\*)** and **SUM(qty)**. Since the measures computed are assumed to be the same, each point in the figure is annotated simply by the group-by attributes. Thus, the point **(storeID, itemID)** represents the cube view corresponding to the query

```
(SI):  SELECT storeID, itemID, COUNT(*), SUM(qty)
        FROM pos
        GROUP BY storeID, itemID .
```



Edges in a lattice run from the node above to the node below. Each edge  $v_1 \rightarrow v_2$  implies that  $v_2$  can be answered using  $v_1$ , instead of accessing the base data. The edge defines a query that derives view  $v_2$  below

from the view  $v1$  above by simply replacing the table in the **FROM** clause with the name of the view above, and by replacing any **COUNT** aggregate function with the **SUM** aggregate function. For example, the edge from  $v1 = (\text{storeID}, \text{itemID}, \text{date})$  to  $v2 = (\text{storeID}, \text{itemID})$  defines the following query equivalent to query  $SI$  above (assuming that the aggregate columns in the views are named `count` and `qty`).

```
(SI'):  SELECT storeID, itemID, SUM(count), SUM(qty)
        FROM v1
        GROUP BY storeID, itemID .
```

**Generalized Cube Views:** However, in most warehouses and decision support systems, the set of summary tables do not fit into the structure of cube views—they differ in their aggregation functions and the joins they perform with the fact tables<sup>1</sup>. Further, some views may do aggregation on columns used as dimension attributes in other views. We will call these views *generalized cube views*, and define them as traditional cube-style views that are extended in the following ways:

- different views may compute different aggregate functions,
- some views may compute aggregate functions over attributes that are used as group-by attributes in other views,
- views may join with different combinations of dimension tables (note that dimension-table joins are always along foreign keys).

### 3.3 Dimension Hierarchies and Lattices

As mentioned in Section 2, the various dimensions represented by the group-by attributes of a fact table often are organized into dimension hierarchies. For example, in the stores dimension, stores can be grouped into cities, and cities can be grouped into regions. In the items dimension, items can be grouped into categories.

The dimension hierarchy information can be stored in separate dimension tables, as we did in the `stores` and `items` tables. In order to group by attributes further along the dimension hierarchy, the fact table must be joined with the dimension tables before doing the aggregation. The joins between the fact table and dimension tables are always along foreign keys, so each tuple in the fact table is guaranteed to join with one and only one tuple from each dimension table.

A dimension hierarchy can also be represented by a lattice, similar to a data-cube lattice. For example, Figure 4(b) shows the lattices for the store and item dimension hierarchies. Note that the bottom element of each lattice is “none,” meaning no grouping by that dimension. Furthermore, although the store and item dimensions depicted here are total orders, partial orders where some elements in the hierarchy are incomparable are also possible—such as in the time dimension, where weeks and months do not strictly contain each other.

We can construct a lattice representing the set of views that can be obtained by grouping on each combination of elements from the set of dimension hierarchies. It turns out that a *direct product* of the the lattice for the fact table along with the lattices for the dimension hierarchies yields the desired result [HRU96]. For example, Figure 5 shows the lattice combining the fact table lattice of Figure 4(a) with the dimension hierarchy lattices of Figure 4(b) (ignore the edge annotations for now).

### 3.4 Partial Lattices

A partial lattice is obtained by removing some nodes of the lattice, to represent the fact that the corresponding views are not being materialized. When a node  $n$  is removed, all incoming and outgoing edges from node  $n$  are also removed, and new edges are added between nodes above and below node  $n$ . For every incoming edge

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<sup>1</sup>They may also differ in the **WHERE** clause, but we do not consider differing **WHERE** clauses in this paper.

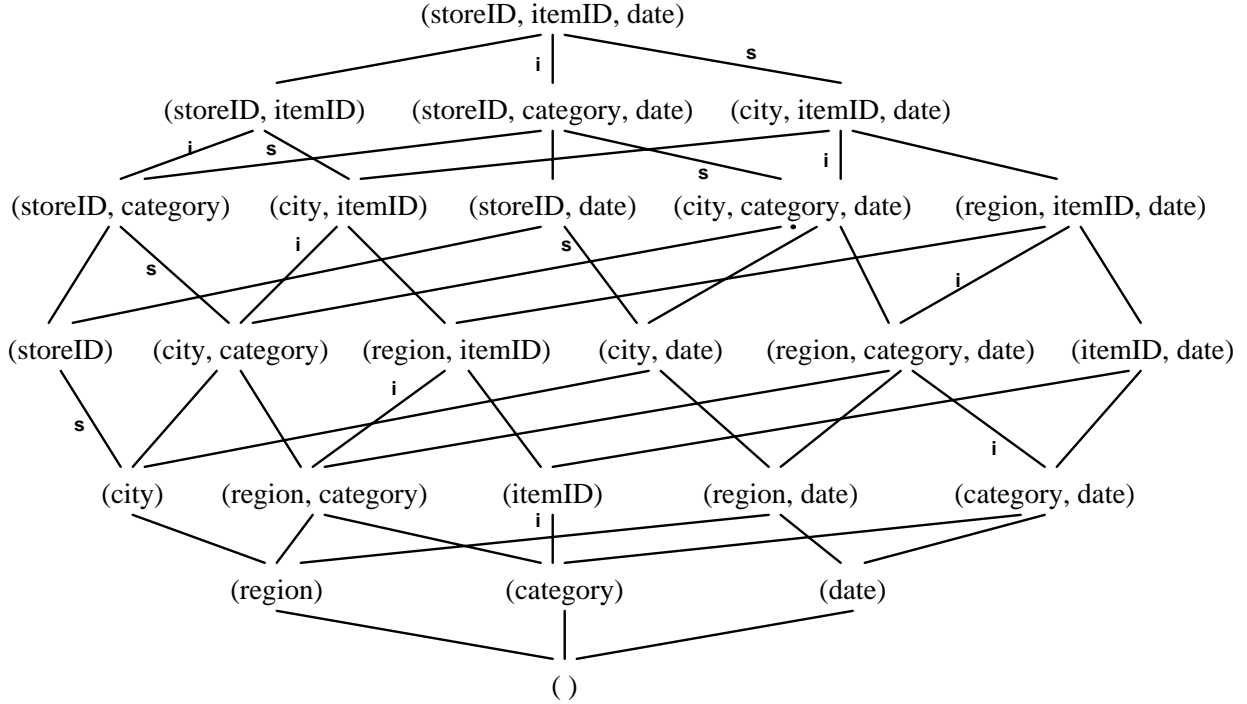


Figure 5: Combined lattice.

$(n1, n)$ , and every outgoing edge  $(n, n2)$ , we add an edge  $(n1, n2)$ . The query defining view  $n2$  along the edge  $(n1, n2)$  is obtained from the query along the edge  $(n, n2)$  by replacing view  $n$  in the **FROM** clause with view  $n1$ . Note that if the top and/or bottom elements of the lattice are removed, the resulting partial lattice may not be a lattice - it represents a partial order between nodes without a top and/or a bottom element.

## 4 Basic summary-delta maintenance algorithm

In this section we show how to efficiently maintain a summary table given changes to the base data. Specifically, we give propagate and refresh functions for maintaining a generalized cube view of the type described in Section 3.2, including joins with dimension tables. We require that the aggregate functions calculated in the summary table either be self-maintainable, be made self-maintainable by adding the appropriate **COUNT** functions as described in Section 3.1, or be **MIN** or **MAX** aggregate functions (in which case the circumstances under which they are not self-maintainable are detected and handled in the refresh function). For simplicity, we start out by considering changes (insertions and deletions) only to the base fact table. We consider changes to the dimension tables in Section 4.1.4.

### 4.1 Propagate Function

As described briefly in Section 2, the general intuition for the propagate function is to create a *summary-delta table* that contains the net effect of the changes on the summary table. Since the propagate function does not affect the summary table, the summary table can continue to be available to readers while the propagate function is computed. Therefore, the goal of the propagate function is to do as much work as possible so that the time required by the refresh function is minimized.

### 4.1.1 Preparing changes

In order to make the computation of the summary-delta table easier to understand, we split up some of the work by first defining three *virtual* views: *prepare-changes*, *prepare-insertions* and *prepare-deletions*. The *prepare-changes* virtual view is defined simply as the union of *prepare-insertions* and *prepare-deletions*, which are described below. In Section 4.1.2 we will see that the summary-delta table is computed from the *prepare-changes* virtual view.

The *prepare-insertions* and *prepare-deletions* virtual views return the changes to the aggregate functions caused by individual insertions and deletions, respectively, to the base data. They take a projection of the insertions/deletions to the base data, after applying any selections conditions and joins that appear in the definition of the summary table. The projected attributes include

- each of the group-by attributes of the summary table, and
- *aggregate-source* attributes corresponding to each of the aggregate functions computed in the summary table.

An *aggregate-source* attribute computes the result of the expression on which the aggregate function is applied. For example, if the summary table included the aggregate function  $\text{sum}(A*B)$ , the *prepare-insertions* and *prepare-deletions* virtual views would each include in their select clause an *aggregate-source* attribute computing  $A * B$  (*prepare-deletions* actually includes  $-(A * B)$ ). We will see later that when the summary-delta table is computed, the *aggregate-source* attributes are aggregated.

The *aggregate-source* attributes are derived according to Table 2. The column labeled *prepare-insertions* describes how they are derived for the *prepare-insertions* view; the column labeled *prepare-deletions* describes how they are derived for the *prepare-deletions* view. The  $\text{COUNT}(expr)$  row uses the SQL-92 case statement [MS93].

	prepare-insertions	prepare-deletions
$\text{COUNT}(*)$	1	-1
$\text{COUNT}(expr)$	case when $expr$ is null then 0 else 1 end	case when $expr$ is null then 0 else -1 end
$\text{SUM}(expr)$	$expr$	$-expr$
$\text{MIN}(expr)$	$expr$	$expr$
$\text{MAX}(expr)$	$expr$	$expr$

Table 2: Deriving *aggregate-source* attributes

**EXAMPLE 4.1** Consider the `SiC_sales` view of Figure 1. The *prepare-insertions*, *prepare-deletions*, and *prepare-changes* virtual views for `SiC_sales` are shown in Figure 6. The *prepare-insertions* view is prefixed by “`pi_`,” *prepare-deletions* is prefixed by “`pd_`,” and *prepare-changes* is prefixed by “`pc_`.” The *aggregate sources* are named `_count`, `_date`, and `_quantity`, respectively. ■

### 4.1.2 Computing the summary-delta table

The summary-delta table is computed by aggregating the *prepare-changes* virtual view. The summary-delta table has the same schema as the summary table, except that the attributes resulting from aggregate functions in the summary delta represent changes to the corresponding aggregate functions in the summary table. For this reason we name attributes resulting from aggregate functions in the summary-delta table as the name of the corresponding attribute in the summary table, prefixed by “`sd_`”

Each tuple in the summary-delta table describes the effect of the base-data changes on the aggregate functions of a corresponding tuple in the summary table (*i.e.*, a tuple in the summary table having the same

```

CREATE VIEW pi_SiC_sales(storeID, category, _count, _date, _quantity) AS
SELECT storeID, category, 1 AS _count, date AS _date, qty AS _quantity
FROM pos_ins, items
WHERE pos_ins.itemID = items.itemID

CREATE VIEW pd_SiC_sales(storeID, category, _count, _date, _quantity) AS
SELECT storeID, category, -1 AS _count, date AS _date, -qty AS _quantity
FROM pos_del, items
WHERE pos_del.itemID = items.itemID

CREATE VIEW pc_SiC_sales(storeID, category, _count, _date, _quantity) AS
  SELECT *
  FROM (pi_SiC_sales UNION ALL pd_SiC_sales)

```

Figure 6: Prepare changes example

values for all group-by attributes as the tuple in the summary-delta table). Note that a corresponding tuple in the summary table may not exist, and in fact it is sometimes necessary in the refresh function to insert a tuple into (or delete a tuple from) the summary table due to the changes represented in the summary-delta table.

The query to compute the summary-delta table follows from the query computing the summary table, with the following differences:

- The from clause is replaced by prepare-changes.
- The where clause is removed. (It is already applied in the definition of prepare-insertions and prepare-deletions.)
- The expressions on which the aggregate functions are applied are replaced by references to the aggregate-source attributes in prepare-changes.
- **COUNT** aggregate functions are replaced by **SUM**.

**EXAMPLE 4.2** Consider again the `SiC_sales` view of Figure 1. The query computing the summary-delta table for `SiC_sales` is shown below. It aggregates the changes represented in the prepare-changes virtual view, grouping by the same group-by attributes as the summary table.

```

CREATE VIEW sd_SiC_sales(storeID, category, sd_Count, sd_EarliestSale, sd_Quantity) AS
SELECT storeID, category, sum(_count) AS sd_Count, min(_date) AS sd_EarliestSale, sum(_quantity) AS sd_Quantity
FROM pc_SiC_sales
GROUP BY storeID, category

```

The astute reader will recall that in Section 2.2 the summary-delta table for `SiC_sales` was defined using the summary-delta table for `SID_sales`. In this example we defined the summary-delta table using instead the changes to the base data. ■

### 4.1.3 Pre-aggregation

As a potential optimization, it is possible to pre-aggregate the insertions and deletions before joining with some of the dimension tables. In particular, joins with dimension tables whose attributes are not referenced in the aggregate functions, can be delayed until after pre-aggregation. Delaying joins until after pre-aggregation reduces the number of tuples involved in the join, potentially speeding up the computation of the summary-delta table. The decision of whether or not to pre-aggregate could be made in a cost-based manner by a query

optimizer. The notion of pre-aggregation presented here follows essentially from the idea of pushing down aggregation presented in [CS94, GHQ95, YL95].

We illustrate the notion of pre-aggregation by changing the definition of prepare-changes, prepare-insertions, and prepare-deletions, along with the summary-delta table computation as follows. Joins with dimension tables whose attributes are not referenced in the aggregate functions are moved from prepare-insertions and prepare-deletions to the summary-delta table computation. An aggregation step is added to prepare-changes: The aggregate functions are identical to those computed by the summary-delta table; The group-by attributes are the ones in the summary-delta table, less group-by attributes coming from dimension tables that have been moved up to the summary-delta computation, plus fact-table attributes that are used to join into the dimension tables that have been moved up to the summary-delta computation. Finally, in the definition of the summary-delta table, if we need to compute  $\text{SUM}(A)$  or  $\text{COUNT}(A)$  on an attribute  $A$  that becomes a group-by attribute in prepare-changes, we need to use  $\text{SUM}(A * Y)$  and  $\text{SUM}(Y)$  respectively, where  $Y$  is the result of  $\text{COUNT}(*)$  in prepare-changes.

```

CREATE VIEW pi_SiC_sales(storeID, category, _count, _date, _quantity) AS
SELECT storeID, category, 1 AS _count, date AS _date, qty AS _quantity
FROM pos_ins

CREATE VIEW pd_SiC_sales(storeID, category, _count, _date, _quantity) AS
SELECT storeID, category, -1 AS _count, date AS _date, -qty AS _quantity
FROM pos_del

CREATE VIEW pc_SiC_sales(storeID, category, _count, _date, _quantity) AS
SELECT storeID, category, sum(_count) AS _count, min(_date) AS _date, sum(_quantity) AS _quantity
FROM (pi_SiC_sales UNION ALL pd_SiC_sales)
GROUP BY storeID, itemID

CREATE VIEW sd_SiC_sales(storeID, category, sd_Count, sd_EarliestSale, sd_Quantity) AS
SELECT storeID, category, sum(_count) AS sd_Count, min(_date) AS sd_EarliestSale, sum(_quantity) AS sd_Quantity
FROM pc_SiC_sales, items
WHERE pc_SiC_sales.itemID = items.itemID
GROUP BY storeID, category

```

Figure 7: Pre-aggregation example

**EXAMPLE 4.3** It is possible to apply pre-aggregation to the summary-delta table computation for `SiC_sales`. Since none of the attributes in the `items` dimension table are referenced in the aggregate functions, the join with `items` can be moved up into the summary-delta table computation. The modified definitions for prepare-insertions, prepare-deletions, prepare-changes, and the summary-delta are shown in Figure 7. ■

#### 4.1.4 Changes to dimension tables

Up to now we have considered changes only to the fact table. Changes also to the dimension tables can be easily incorporated into our method. Due to space constraints we will only give the intuition underlying the technique.

Using the incremental view-maintenance techniques of [GMS93, GL95], we can start with the changes to a dimension table, and derive dimension-table-specific prepare-insertions and prepare-deletions views that describe the tuples being inserted into and deleted into a table that gets aggregated. to a dimension table. For example, the following view definition calculates prepare-insertions for `SiC_sales` due to insertions to `items` (made available in `items_ins`).

```

CREATE VIEW pi_items_SiC_sales(storeID, category, _count, _date, _quantity) AS
SELECT storeID, category, 1 AS _count, date AS _date, qty AS _quantity
FROM pos, items_ins
WHERE pos.itemID = items_ins.itemID

```

Prepare-changes then takes the union of all insertions and deletions to the set of tuples to aggregate due to changes to the fact table and all dimension tables, and the summary-delta computation proceeds as before.

## 4.2 Refresh Function

The refresh function applies the changes represented in the summary-delta table to the summary table. Each tuple in the summary-delta table causes a change to a single corresponding tuple in the summary table (by corresponding we mean a tuple in the summary table having the same values for all group-by attributes as the tuple in the summary delta). The corresponding tuple in the summary table is either updated, deleted, or if the corresponding tuple is not found, the summary-delta tuple is inserted into the summary table.

The refresh algorithm is shown in Figure 8. It generalizes and extends the example refresh function given in

```

For each tuple  $\delta t$  in the summary-delta table,
  % get the corresponding tuple in the summary table
  Let tuple  $t$  = tuple in the summary table having the same values
    for its group-by attributes as  $\delta t$ 
  If  $t$  is not found,
    % insert tuple
    Insert tuple  $\delta t$  into the summary table
  Else
    % check if the tuple needs to be deleted
    If  $\delta t.COUNT(*) + t.COUNT(*) = 0$ ,
      Delete tuple  $t$ 
    Else
      % check if the min/max values needs to be recomputed
      recompute = false
      For each MIN and MAX aggregate function  $m(e)$  in the summary table,
        If (( $m$  is a MIN function AND  $\delta t.MIN(e) \leq t.MIN(e)$  AND  $t.COUNT(e) + \delta t.COUNT(e) > 0$ ) OR
            ( $m$  is a MAX function AND  $\delta t.MAX(e) \geq t.MAX(e)$  AND  $t.COUNT(e) + \delta t.COUNT(e) > 0$ ))
          recompute = true
      If (recompute) % min/max values of tuple  $t$  needs to be recomputed
        Update tuple  $t$  by recomputing its aggregation functions from the base data for  $t$ 's group.
      Else
        % update the tuple
        For each aggregate function  $a(e)$  in the summary table,
          If  $t.COUNT(e) + \delta t.COUNT(e) = 0$ ,
             $t.a = \text{null}$ 
          Else If  $a$  is COUNT or SUM,
             $t.a = t.a + \delta t.a$ 
          Else If  $a$  is MIN,
             $t.a = \text{MIN}(t.a, \delta t.a)$ 
          Else If  $a$  is MAX,
             $t.a = \text{MAX}(t.a, \delta t.a)$ 

```

Figure 8: The Refresh function

Section 2.1, by handling the case of nulls in the input and **MIN** and **MAX** aggregate functions. In the algorithm,

for each tuple  $\delta t$  in the summary-delta table the corresponding tuple  $t$  in the summary table is looked up. If  $t$  is not found, the summary-delta tuple is inserted into the summary table. If  $t$  is found, then if  $\text{COUNT}^*$  from  $t$  plus  $\text{COUNT}^*$  from  $\delta t$  is zero, then  $t$  is deleted.<sup>2</sup> Otherwise, a check is performed for each of the **MIN** and **MAX** aggregate functions, to see if a value less than or equal to the minimum (greater than or equal to the maximum) value was deleted, in which case the new **MIN** or **MAX** value of  $t$  will probably need to be recomputed. The only exception is if  $\text{COUNT}(e)$  from  $t$  plus  $\text{COUNT}(e)$  from  $\delta t$  is zero, in which case the new min/max/sum/count( $e$ ) values are null.

As the last step in the algorithm, the aggregation functions of tuple  $t$  are updated from the values in  $\delta t$ , or (if needed) by recomputing a min/max value from the base data for  $t$ 's group. For simplicity in the recomputation, we assume that when a summary table is being refreshed, the changes have already been applied to the base data. However, an alternative would be to do the recomputation before the changes have been applied to the base table by issuing a query that subtracts the deletions from the base data and unions the insertions. As written, the refresh function only considers the **COUNT**, **SUM**, **MIN**, and **MAX** aggregate functions, but it should be easy to see how any self-maintainable aggregation function would be incorporated.

The above refresh function may appear complex, but conceptually it is very simple. One can think of it as an outer-join between the summary-delta table and the summary table. Each summary table tuple that joins with a summary-delta tuple is updated or deleted as it joins, while a summary-delta tuple that does not join is inserted into the summary table. The only complication in the process is an occasional recomputation of a min/max value. Such a *summary-delta join* needs to be implemented in the database server, and should be implemented by database vendors that are targeting the warehouse market.

### 4.3 Optimization of the refresh function

It is possible to speed up the refresh function of Figure 8 if certain integrity constraints are known to hold on the changes.

First, if the changes are known to contain only inserted tuples, the checks to see if tuple  $t$  needs to be deleted or recomputed need not be performed. Either  $\delta t$  is inserted into the summary table if  $t$  is not found, or  $t$  is updated from the existing value of  $t$  and  $\delta t$ .

Second, if it is known that tuples in the changes have different values for at least one of the group-by attributes of the summary table than existing tuples in the summary table, then the query to locate the corresponding tuple in the summary table need not be performed. Instead, tuples in the summary-delta table are simply inserted into the summary table. For example, if it is known that base-data changes are always for a new date, then tuples in `sd_SID_sales` would simply be inserted into `SID_sales`. This optimization can lead to a significant reduction in the refresh times. The performance graphs (comparing Figure 10(a) with Figure 10(c) and Figure 10(b) with Figure 10(d)) illustrate the savings due to this optimization.

Along the same line, if it is known that tuples in the summary-delta table *usually* have different values for at least one of the group-by attributes, and a unique key is declared on the group-by attributes of the summary table, then it is probably more efficient to attempt to insert tuples from the summary-delta table into the summary table straightaway. If a unique key conflict is generated due to the insertion, then the corresponding tuple in the summary table can instead be fetched and updated.

Finally, if it is known that the changes are *weakly minimal* [GL95], meaning that the set of deleted tuples is guaranteed to be a subset of the existing base-data tuples, or in other words, that a tuple cannot be inserted and subsequently deleted in the same batch of changes, then the recomputation test in the refresh function can be made tighter. Specifically, we can change the  $\delta t.m \leq t.m$  and  $\delta t.m \geq t.m$  tests to the more restrictive test  $\delta t.m = t.m$ . To explain, for a **MIN** aggregate function, if  $\delta t.m < t.m$  then we know that a tuple must have been inserted with a lower value for  $m$  than existing tuples in the summary table. Since inserted tuples

---

<sup>2</sup>Note that  $\text{COUNT}^*$  from  $t$  plus  $\text{COUNT}^*$  from  $\delta t$  can never be less than zero, and that if  $\text{COUNT}^*$  from  $\delta t$  is less than zero, then the corresponding tuples  $t$  must be found in the summary table.



cannot be deleted due to the weak minimality constraint,  $\delta t.m$  must be the new minimum value; therefore, recomputation is not necessary. A similar argument holds for **MAX**.

## 5 Efficiently maintaining multiple summary tables

In the previous section we have shown how to compute the summary-delta table for a generalized cube view, directly from the insertions and deletions into the base fact table.

We have also seen that multiple cube views can be arranged into a (partial) lattice (Section 3.3). We will show that multiple summary tables, which are generalized cube views, can also be placed in a (partial) lattice, which we call a  $\mathcal{V}$ -lattice. Further, all the summary-delta tables can also be written as generalized cube views, and can be placed in a (partial) lattice, which we call a  $\mathcal{D}$ -lattice. It turns out that the  $\mathcal{D}$ -lattice is identical to the  $\mathcal{V}$ -lattice, modulo renaming of tables.

### 5.1 Placing Generalized Cube Views into a Lattice

The principal behind the placement of cube views in a lattice is that a cube view  $v_2$  should be derivable from the cube view  $v_1$  placed above  $v_2$  in the cube lattice. The same principle can be adapted to place a given set of generalized cube views into a (partial) lattice. We will show how to define a *derives relation*  $v_2 \prec v_1$  between the given set of generalized cube views.  $\prec$  can be used to impose a partial ordering on the set of generalized views, and to place the views into a (partial) lattice, with  $v_1$  being an ancestor of  $v_2$  in the lattice if and only if  $v_2 \prec v_1$ .

For two generalized cube views  $v_1$  and  $v_2$ , let  $v_2 \prec v_1$  if and only if view  $v_2$  can be defined using a single block **SELECT -FROM -GROUP BY** query over view  $v_1$  possibly joined with one or more dimension tables on the foreign key. The  $v_2 \prec v_1$  condition holds if

1. each group-by attribute of  $v_2$  is either a groupby attribute of  $v_1$ , or is an attribute of a dimension table whose foreign key is a groupby attribute of  $v_1$ , and
2. each aggregate function  $a(E)$  of  $v_2$  either appears in  $v_1$ , or  $E$  is an expression over the groupby attributes of  $v_1$ , or  $E$  is an expression over attributes of dimension tables whose foreign keys are groupby attributes of  $v_1$ .

If the above conditions are satisfied using dimension tables  $d_1, \dots, d_m$ , we will superscript the  $\prec$  relation as  $\prec^{d_1, \dots, d_m}$ .

**EXAMPLE 5.1** For our running retailing warehouse example, we can derive the following precedence relations:  $sCD\_sales \prec^{stores} SID\_sales$ ,  $SiC\_sales \prec^{items} SID\_sales$ ,  $sR\_sales \prec^{stores} SID\_sales$ ,  $sR\_sales \prec^{stores} sCD\_sales$ , and  $sR\_sales \prec^{stores} SiC\_sales$ .  $SID\_sales$  is the top and  $sR\_sales$  is the bottom of the lattice. ■

The query associated with an edge from  $v_1$  to  $v_2$  is obtained from the original query for  $v_2$  by making the following changes:

- The original **WHERE** clause is removed (it is not needed since the conditions already appear in  $v_1$ ).
- The **FROM** clause is replaced by a reference to  $v_1$ . Further, if the  $\prec$  relation between  $v_2$  and  $v_1$  is superscripted with dimension tables, these are joined into  $v_1$ . (The dimension tables will go into the **FROM** clause, and the join conditions will go into the **WHERE** clause.)
- The aggregate functions of  $v_2$  need to be rewritten to reference the aggregate function results computed in  $v_1$ . In particular,
  - A **COUNT** aggregate function needs to be changed to a **SUM** of the counts computed in  $v_1$ .
  - If  $v_1$  groups by an attribute  $A$  and  $v_2$  computes **SUM**( $A$ ), then **SUM**( $A$ ) will be replaced by **SUM**( $A*Y$ ), where  $Y$  is the result of **COUNT**(\*) in  $v_1$ . Similarly **COUNT**( $A$ ) will be replaced by **SUM**( $Y$ ).

## 5.2 Making Summary Tables Lattice-Friendly

It is also possible to change the definitions of summary tables slightly so that the derives relation between them grows larger, and we do not repeat joins along the lattice paths. The summary tables are changed by adding joins with dimension tables, adding dimension attributes, and adding aggregation functions used by other summary tables.

Let us consider the case of dimension tables and dimension attributes. Are joins with dimension tables all performed implicitly at the top-most view, or could they be performed lower down just before grouping by dimension attributes? Because the joins between the fact table and the dimension tables are along foreign keys—so that each tuple in the fact table joins with one and only one tuple from each dimension table—either approach, joining implicitly at the top-most view or just before grouping on dimension attributes, is possible.

Now, consider a dimension hierarchy. An attribute in the hierarchy functionally determines all of its descendents in the hierarchy. Therefore, grouping by an attribute in the hierarchy yields the same groups as grouping by that attribute plus all of its descendent attributes. For example, grouping by (storeID) is the same as grouping by (storeID, city, region).

The above two properties provide the rationale for our approach: joining the fact table with all dimension tables at the top-most point in the lattice. At each point in the lattice, instead of grouping only by the group-by attributes mentioned at that point, we include as well each dimension attribute functionally determined by the group-by attributes. For example, the top-most point in the lattice of Figure 5 groups by (storeID, city, region, itemID, category, date).

The end result of the process can be to fit the generalized views into a regular cube (partial) lattice where all the joins are taken once at the top-most point, and all the views have the same aggregation functions.

**EXAMPLE 5.2** For our running warehousing example, we can define all four summary tables as a groupby over the join of `pos`, `items`, and `stores`, computing `COUNT(*)`, `SUM(qty)`, and `MIN(date)` in each view, and retaining some or all of the dimension attributes City, Region, and Category. The resulting lattice represents a portion of the complete lattice shown in Figure 5. ■

## 5.3 Optimizing the lattice

Although the approach of Section 5.2 is always correct, it does not yield the most efficient result. An important question is where best to do the joins with the dimension tables. Further, assuming that some of the dimension columns and aggregation functions have been added to the views just so that the view fits into the lattice, where should the aggregation functions and the extra columns be computed? Optimizing a lattice means pushing joins, aggregation functions, and projections as low down into the lattice as possible.

There are two reasons for pushing down joins: First, as one travels down the data cube, the number of tuples at each point is likely to decrease, so fewer tuples need to be involved in the join. Second, joining with all dimension tables at the top-most view results in very wide tuples, which require more room in memory and on disk. For example, when computing the data cube in Figure 5, instead of joining the `pos` table with `stores` and `items` to compute the (storeID, itemID, date) view, it may be better to push down the join with `stores` until the (city, itemID, date) view is computed from the (storeID, itemID, date) view, and to push down the join with `items` until the (storeID, category, date) view is computed from the (storeID, itemID, date) view.

**EXAMPLE 5.3** Applying the above reasoning to the lattice of Figure 5, we have pushed each join as far down as possible. The top (storeID, itemID, date) is defined without any joins with the dimension tables. The lattice edges are labeled with the dimension join required when deriving the lower view. An edge from  $v_1$  to  $v_2$  is labeled  $s$  if  $v_1$  needs to be joined with `stores` to derive  $v_2$ . An edge labeled  $i$  implies a join with the `items` table. An unlabeled edge implies that  $v_2$  can be obtained from  $v_1$  without any joins. ■

**EXAMPLE 5.4** For the running retail warehousing example, optimization derives the lattice shown in Figure 9. The view `sCD_sales` is extended by adding the region attribute so that the view `sR_sales` may be derived

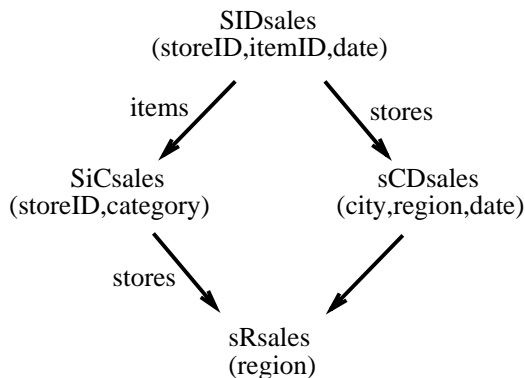


Figure 9: The  $\mathcal{V}$ -lattice for the retail warehousing example

from it without (re-)joining with the `stores` table. ■

Appendix B gives an algorithm to optimize a lattice by pushing down joins, aggregations, and projections as much as possible. The queries along each edge of the resulting lattice are derived as explained at the end of Section 5.1.

## 5.4 Summary-Delta Lattice

Following the self-maintenance conditions discussed in Section 3.1, we assume that any view computing an aggregation function is augmented with `COUNT( $\star$ )`. A view computing `SUM( $E$ )`, `MIN( $E$ )`, and/or `MAX( $E$ )` is further augmented with `COUNT( $E$ )`.

Given the set of generalized cube views in the partial  $\mathcal{V}$  lattice, we would like to arrange the summary-delta tables for these views into a partial lattice (the  $\mathcal{D}$  lattice). The hope is that we can then compute the summary-delta tables more efficiently by exploiting the  $\mathcal{D}$  lattice structure, just as the views can be computed more efficiently by exploiting the  $\mathcal{V}$  lattice structure.

The following theorem follows from the observation that the query defining each a summary-delta table  $sd_v$  (Section 4.1) is similar to the query defining the view  $v$ , except that some of the tables in the `FROM` clause are uniformly replaced by the `prepare-changes` table. The theorem gives us the desired  $\mathcal{D}$ -lattice.

**Theorem 5.1** *The  $\mathcal{D}$ -lattice is identical to the  $\mathcal{V}$ -lattice, including the join annotations and the queries along each edge, modulo a change in the name of table.* ■

Thus, each summary delta table can be derived from the summary-delta table above it in the partial lattice by a join with any annotated dimension table, followed by a simple groupby operation. The queries defining the topmost summary-delta tables in the  $\mathcal{D}$ -lattice are obtained by defining a `prepare-changes` virtual view (Section 4.1).

**EXAMPLE 5.5** The summary-delta  $\mathcal{D}$ -lattice for our warehouse example is the same as the partial  $\mathcal{V}$ -lattice of Figure 9. The query defining `sd_SID_sales` in terms of `prepare-changes` appears in Section 2.1, where the union subquery in the `FROM` clause should be interpreted as the `prepare-changes` view. The queries defining the summary-delta tables along three of the edges of the lattice are the same as those in Figure 3. For the fourth edge from `sd_SiC_sales` to `sd_sR_sales`, the query is

```

CREATE VIEW sd_sR_sales(region, sd_Count, sd_Quantity) AS
SELECT region, sum(sd_Count) AS sd_Count, sum(sd_Quantity) AS sd_Quantity
FROM sd_SiC_sales, stores
  
```

```
WHERE sd_SiC_sales.storeID = stores.storeID
GROUP BY region
```



## 5.5 Computing the summary-delta lattice

The beauty of our approach is that the summary table maintenance problem has been partitioned into two subproblems — computation of summary-delta tables (propagation), and the application of refresh functions — in such a way that the subproblem of propagation for multiple summary tables can be mapped to the problem of efficiently computing multiple aggregate views in a lattice.

Propagation of changes to multiple summary tables involves computing all the summary-delta tables in the  $\mathcal{D}$ -lattice derived in Section 5.4. The problem now is how to compute the summary-delta lattice efficiently, since there are possibly several choices for ancestor summary-delta tables from which to compute a summary-delta. It turns out that that this problem maps directly to the problem of computing multiple summary tables from scratch, as addressed in [AAD<sup>+</sup>96, SAG96]. We can use their solutions to derive an efficient propagate strategy on how to sort/hash inputs, what order to evaluate summary-delta tables, and which of the incoming lattice edges (if there is more than one) to use to evaluate a summary-delta table. The algorithms of [AAD<sup>+</sup>96, SAG96] would be directly applicable but for the fact that they do not consider join annotations in the lattice. However, it is a simple matter to extend their algorithms by including the join cost estimate in the cost of the derivation of the aggregate view along the edge annotated with the join. We omit the details here as the algorithms for materializing a lattice are not the focus of this paper.

## 6 Performance

We have implemented the summary-delta algorithm on top of a common PC-based relational database system. We have used the implementation to test the performance improvements obtained by the summary-delta table method over recomputation and the counting method, and to determine the marginal gains to the propagate function from exploiting the lattice structure when maintaining multiple summary tables.

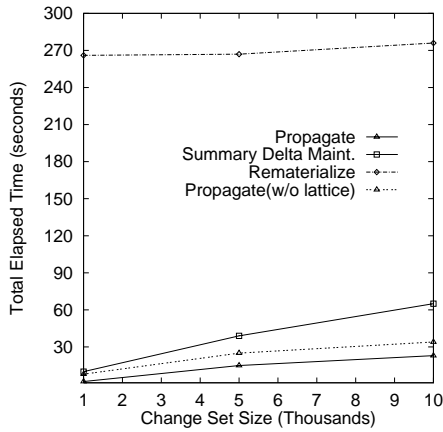
The implementation was done in Centura SQL Application Language (SAL) on a Pentium PC. The test database schema is the same as the one used in our running example described in Section 2. We varied the size of the `pos` table from 100,000 tuples to 500,000 tuples, and the size of the changes from 1,000 tuples to 10,000 tuples. The `pos` table had a composite index on (storeID, itemID, date), and each of the summary tables had composite indices on their groupby columns. We found that the performance of the refresh operation depended heavily on the number of updates/deletes vs inserts to the summary tables. Consequently, we considered two types of changes to the `pos` table:

- **Update-Generating Changes:** Insertions and deletions of an equal number of tuples over existing date, store, and item values. These changes mostly cause updates amongst the existing tuples in summary-delta tables.
- **Insertion-Generating Changes:** Insertions over new dates, but existing store and item values. These changes cause only inserts into two of the four summary-delta (for whom date is a groupby column), and mostly cause updates into the other two summary-delta tables.

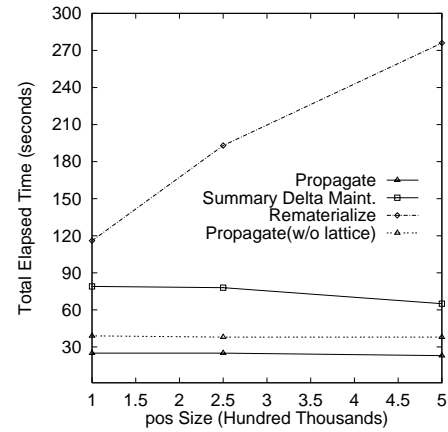
The insertion-generating changes are very meaningful since in many data warehousing applications the only changes to the fact tables are insertions of tuples for new dates, which leads to insertions, but no updates, into summary tables with date as a groupby column.

Figure 10 shows four graphs illustrating the performance advantage of using the summary-delta table method. The graphs show the time to rematerialize (using the lattice structure), and maintain all four summary tables using the summary-delta table method (using the lattice structure). The maintenance time is split into propagate and refresh, with the lower solid line representing the portion of the maintenance time

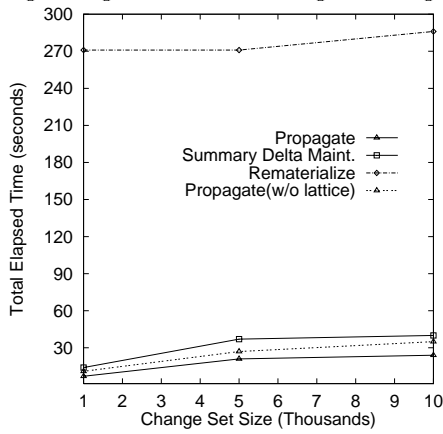
taken by propagate when using the lattice structure. The upper solid line represents the total maintenance time (propagate + refresh). The time taken by propagate without using the lattice structure is shown with a dotted line for comparison.



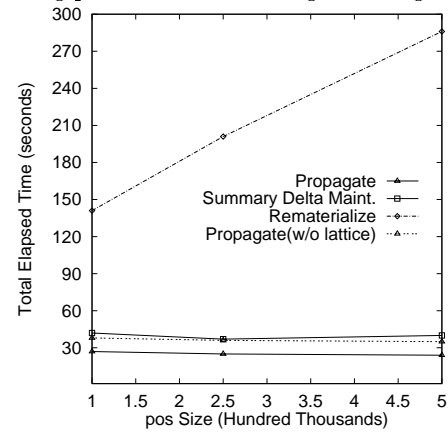
(a) Varying change size for update-generating changes



(b) Varying pos size for update-generating changes



(c) Varying change size for insertion-generating changes



(d) Varying pos size for insertion-generating changes

Figure 10: Performance of Summary-Delta Maintenance algorithm

Graphs 10(a) and 10(c) plot the variation in elapsed time as the size of the change set changes, for a fixed size (500,000) of the `pos` table. While 10(a) considers update-generating changes, graph 10(c) considers insertion-generating changes. We note that the incremental maintenance wins for both types of changes, but it wins with a far greater margin for the insertion-generating changes. The difference between the two scenarios is mainly in the refresh times for the views `SID_sales` and `sCD_sales`; The refresh time going down by 50% in 10(c). The graphs also show that the summary-delta maintenance beats rematerialization, and that propagate benefits by exploiting the lattice structure. Further, the benefit to propagate increases as the size of the change set increases.

Graphs 10(b) and 10(d) plot the variation in elapsed time as the size of the `pos` table changes, for a fixed size (10,000) of the change set. Graph 10(b) considers update generating changes, and graph 10(d) considers insertion generating changes. We see that the propagate time stays virtually constant with increase in the size of `pos` table (as one would expect, since propagate does not depend on the `pos` table); However interestingly the refresh time goes down for the update generating changes. A close look reveals that when the `pos` table is small, refresh causes a significant number deletions in addition to updates to the materialized views. When the `pos` table is large, refresh causes only updates to the materialized views, and this leads to a 20% savings

in refresh time.

**Counting Algorithm** The counting algorithm applied to summary tables turned out to be a big disappointment. We found it to be uniformly worse than recomputation by one order of magnitude for the update generating changes, and worse than summary-delta algorithm by a factor of 1 – 2 for the insertion generating changes, even on the views with date as a groupby column. The reason is that the counting algorithm causes a significant fraction of the summary table to be deleted, and then recomputed using expensive accesses to the `pos` table.

## 7 Related Work and Conclusions

Both view maintenance and data warehousing are active areas of research, and this paper is in the intersection of the two areas, proposing new view maintenance techniques for maintaining multiple summary tables (aggregate views) over a star schema using a new *summary-delta* paradigm.

Earlier papers on view maintenance [BLT86, CW91, QW91, GMS93, GL95, JMS95, ZGHW95, CGL<sup>+</sup>96, HZ96, Qua96] have all used the *delta* paradigm - compute a set of inserted and deleted tuples that are then used to refresh the materialized view using simple union and difference operations. The new summary-delta paradigm is to compute a summary-delta table that represents a summary of the changes to be applied to the materialized view. The actual refresh of the materialized view is more complex than a union/difference in the delta paradigm, and can cause updates, insertions, and/or deletions to the materialized view. Amongst the above work on view maintenance algorithm, [GMS93, GL95, JMS95, Qua96] are the only papers that discuss maintenance algorithms for aggregate views. While a detailed comparison of the summary-delta table method against these is made in Section 2.3, it is worth noting that the previous papers do not consider the problem of maintaining multiple aggregate views, and are not as efficient as the summary-delta table method.

A formal split of the maintenance process into propagate and refresh functions was proposed in [CGL<sup>+</sup>96]. We build on the propagate/refresh idea here, extending it to aggregate views and to more complex refresh functions. Our notion of self-maintainable aggregation functions is an extension of self-maintainability for select-project-join views defined in [GJM96, QGMW96].

[GBLP96] proposed the cube operator linking together related aggregate tables into one SQL query, and starting a mini-industry in warehousing research. The notion of cube lattices and dimension lattices was proposed in [HRU96], along with an algorithm to determine a subset of cube views to be materialized so as to maximize the querying benefit under a given space constraint. Algorithms to efficiently materialize all or a subset of the cube lattice have been proposed by [AAD<sup>+</sup>96, SAG96]. Next, we need a technique to maintain these cube views efficiently, and our paper provides the summary-delta table method to do so. In fact, we even map a part of the maintenance problem into the problem addressed by [AAD<sup>+</sup>96, SAG96]

Our algorithms are geared towards cube views, as well as towards generalizations of cube views that are likely to occur in typical decision-support systems. We have developed techniques to place aggregate views into a lattice, even suggesting small modifications to the views that can help generate a fuller lattice.

Finally, we have tested the feasibility and the performance gains of the summary-delta table method by implementing it on top of a relational database, and doing a performance study comparing the propagate and refresh times of our algorithm to the alternatives of doing rematerializations or using an alternative maintenance algorithm. We found that our algorithm provides an order of magnitude improvement over the alternatives. Another observation we made from the performance study is that our refresh function, when implemented outside the database system, runs much slower than what we had expected (while still being fast). The right way to implement the refresh function is by doing something similar to an *outer-join* of the summary-delta table with the materialized view, identifying the view tuples to be updated, and updating them as a part of the join. Such an operation, which we will call a *summary-delta join* should be built into the database servers that are targeting the warehousing market.

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## A Details behind Table 1

In this section we will describe the details of the database and analytical computations used to derive the number of tuple reads and writes in Table 1.

We chose a database with the number of tuples in the **pos** fact table, **stores** and **items** dimension tables, and the four summary tables as given in Table 3. The “# changes” column records the number of changes

Table name	# tuples	# changes	summary-delta table
<b>pos</b>	1,000,000	10,000	
<b>stores</b>	100		
<b>items</b>	1000		
<b>SID_sales</b>	100,000		1,000
<b>sCD_sales</b>	10,000		100
<b>SiC_sales</b>	2,000		1,000
<b>sR_sales</b>	10		10

Table 3: Table statistics

received to the **pos** table, and the “summary-delta table” column records the number of tuples that the changes would cause to be generated in each summary-delta table.

Assuming the statistics in Table 3, Table 1 showed the number of tuple accesses (reads and writes) required to recompute and incrementally maintain each of the summary tables.

When counting the number of accesses needed to recompute a summary table  $s$ , we assume that each tuple in the table used to compute  $s$  must be accessed, possibly joined with dimension tables, and hashed or sorted on the values of its group-by attributes to perform the aggregation. The tuples in  $s$  must also be written to disk.

When maintaining a summary table by using the summary-delta algorithm directly from the changes to the **pos** table, we read each tuple in the changes to the **pos** table, join it with any dimension tables, and hash or sort on its group-by attributes to perform the aggregation. Then, each tuple in the summary-delta table is used in the refresh function to update (or insert or delete) a corresponding tuple in the summary table. The



tuple accesses count the number of tuples read by the propagation function and the number of tuples updated (inserted, deleted) in the refresh function.

When maintaining the four summary tables together, the summary-delta tables created for one summary table are re-used in the creation of summary-delta tables for other summary tables, as shown in Figure 3. The summary-delta table for `SID_sales` is used to create the summary-delta tables for `sd_sCD_sales` and `SiC_sales`, and the summary-delta table for `sd_sCD_sales` is used to create the summary-delta table for `sR_sales`. As in the second column, the tuple accesses count the number of tuples read by the propagation function and the number of tuples updated (or inserted) by the refresh function.

For the counting algorithm applied directly to the changes to the `pos` table, we count the number of delta tuples accessed, the number of tuples deleted in the view, the number of base `pos` tuples accessed to recompute the new tuples to be inserted into the view, and the number of tuples inserted into the view.

## B An Algorithm to Optimize Lattices

We assume we are given a partial cube lattice, with a query defining each edge of the partial lattice, and the original query defining the generalized cube view at each node.

We start by annotating each node  $n$  in the partial lattice as follows:

- Mark each aggregation function in the original definition of the view as essential at node  $n$ .
- If the original view definition has an aggregation function involving an expression using attributes from a dimension table  $d_i$ , mark the table  $d_i$  as essential at node  $n$ .
- If the original view definition has a groupby column  $a$  from a dimension table  $d_i$ , where  $a$  is not available in the fact table, mark the table  $d_i$ , and the attribute  $a$  as essential at node  $n$ .

**Pushing Joins** Consider an edge from view  $v_2$  above to view  $v_1$  below ( $v_1$  is to be derived from view  $v_2$ ).

If a dimension table  $d_i$  is not essential for view  $v_2$ , then it can be dropped from the definition of  $v_2$ , and the edges out of  $v_2$  are labeled with  $d_i$ , meaning that the definition of the query along the edge requires a join of  $v_2$  with the dimension table  $d_i$ .

We use this rule in a single top down traversal of the lattice to push down join annotation as much as possible. As the dimension table is removed from  $v_2$ , the query defining  $v_2$  is modified by removing all groupby columns coming from the dimension table (except that the key of the dimension table that is also available in the fact table is retained), and any aggregation function involving the dimension attributes.

An edge from view  $v_2$  above to view  $v_1$  below is annotated if computing  $v_1$  from  $v_2$  requires dimension hierarchy information not required to be present in  $v_2$ . Specifically, if computing  $v_1$  requires dimension hierarchy information from a dimension table  $d$ , and  $v_2$  does not already contain the information by an earlier join with  $d$ , then  $v_2$  must be joined with  $d$  to compute  $v_1$ .

**Pushing aggregation functions** If an aggregation function is over a groupby column, then it is not essential, and should be removed from the view definition.

Any edge deriving the aggregation function in a lower view must now aggregate over the groupby column. If the aggregation function is `SUM(A)` or `COUNT(A)`, these must be replaced by `SUM(A * Y)` and `SUM(COUNT(Y))` where  $Y$  is the attribute with the `COUNT(*)` value in the higher view.

If an aggregation function is not essential at a node  $n$ , then it is not essential for all its incoming edges. If an aggregation function is not essential for all outgoing edges, then it is not essential at the node, and should be removed from the view definition.

A single breadth first traversal in reverse topological order is enough to prune all non-essential aggregation functions, and rewrite the queries defining the views.

**Pushing Projections** If a (dimension) attribute is not essential at a node  $n$ , it should be removed from node  $n$ , and it is not essential for the incoming edge.

If a (dimension) attribute is not essential for all outgoing edges, then it is not essential at the node, and should be removed from the view definition.

A single breadth first traversal in reverse topological order is enough to project out all non-essential attributes from groupby and **SELECT** clauses, and rewrite the queries defining the views. then it is essential in all its ancestors. This graph traversal can be overlapped with the elimination of aggregation functions.