A Denotational Semantics for the
Starburst Production Rule Language

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Abstract
Researchers often complain that the behavior of database production rules is difficult to
reason about and understand, due in part to the lack of formal declarative semantics. It has
even been claimed that database production rule languages inherently cannot be given declarative
semantics, in contrast to, e.g., deductive database rule languages. In this short paper we
dispute this claim by giving a denotational semantics for the Starburst database production rule
language.

1 Introduction

Production rules in database systems allow specification of data manipulation operations that are
executed automatically whenever certain events occur or conditions are met, e.g. [Han89, MD89,
SIGP90, WF90]. A wide variety of semantics have been proposed for database production rule
languages; see, e.g. [HW93, Sel89]. Most of these semantics are based to some extent on the
recognize-act cycle of the OPS5 production rule language [BFKM85]. In all cases, the semantics
are described informally or, at best, as an algorithm for rule execution. The lack of formal declarative
semantics for database production rule languages has been discussed at some length, particularly in
the context of understanding and reasoning about rule behavior [AWH92, DD91, KdMS92, RL91]. In
response to this problem, some work has been done in extending deductive database rule languages,
which have a clean declarative semantics [CGT90], to include production rule capabilities. In
[RL91], a declarative semantics is given for such an extension, but the extension is not as powerful
as most integrated database production rule languages. In [KdMS92], a more powerful extension
is considered, but no declarative semantics is specified.

In this short paper we give a denotational semantics for the Starburst rule language [WCL91,
WF90], showing that a formal declarative semantics is possible for database production rules.
Although the semantics is (of course) tailored for the Starburst rule language, a similar semantics
should be definable for other similar database rule languages, e.g. [Han89, SIGP90]. In general, a
denotational semantics for a conventional programming language is defined as a meaning function
that takes any program in the language and produces the (input-output) function computed by
that program [Sto77]. Database production rules are processed in response to user modifications on
persistent data, and the effect of rule processing is additional modifications to that data [HW93].
Hence, a denotational semantics for a database production rule language is defined as a meaning
function that takes any set of rules and produces the function that maps a set of modifications and a database state into the new database state that results from processing those rules.

In Section 2 we give an informal description of the Starburst rule language, which serves both to introduce the language and to illustrate the informal style in which such languages (including this one) typically are specified. Section 3 contains the formal denotational semantics, with relevant domains defined in Section 3.1, supporting functions in Section 3.2, and the meaning function in Section 3.3. This semantics is defined under the assumption that rule selection (choosing which rule to consider first when multiple rules are triggered) is deterministic; Section 3.4 modifies the semantics for nondeterministic rule selection.

2 Informal Description of the Starburst Rule Language

We give an informal description of the set-oriented, SQL-based Starburst production rule language. For numerous examples see [WCL91,WF90]. The description given here is similar to that in, e.g. [WCL91]. A more detailed but still mostly informal definition of the language is given in [WF90]. Prior to this paper, the closest to a formal specification is the execution model given in [AWH92] to prove properties of rule analysis.

Starburst production rules are based on the notion of transitions. A transition is a database state change resulting from execution of a sequence of data manipulation operations. Rules consider only the net effect of transitions, meaning that: (1) if a tuple is updated several times, only the composite update is considered; (2) if a tuple is updated then deleted, only the deletion is considered; (3) if a tuple is inserted then updated, this is considered as inserting the updated tuple; (4) if a tuple is inserted then deleted, this is not considered at all.

The syntax for defining a rule is:

```
create rule name on table
when transition predicate
[ if condition ]
then action
[ precedes rule-list ]
[ follows rule-list ]
```

The transition predicate specifies one or more triggering operations on the rule's table: inserted, deleted, or updated($c_1,\ldots,c_n$), where $c_1,\ldots,c_n$ name columns of the rule's table. The rule is triggered by a given transition if at least one of the specified operations occurred in the net effect of the transition. The optional condition specifies an SQL predicate. The action specifies an arbitrary sequence of SQL data manipulation operations to be executed when the rule is triggered and its condition is true. The optional precedes and follows clauses are used to induce a partial ordering on the set of defined rules. If a rule $r_1$ specifies a rule $r_2$ in its precedes list, or if $r_2$ specifies $r_1$ in its follows list, then $r_1$ is higher than $r_2$ in the ordering. (We also say that $r_1$ has precedence or priority over $r_2$.) When no direct or transitive ordering is specified between two rules, their order is arbitrary. Cyclic orderings are not permitted.
A rule's condition and action may refer to the current state of the database through top-level or nested SQL select operations. In addition, rule conditions and actions may refer to transition tables, which are logical tables reflecting the changes to the rule's table that have occurred during the triggering transition. At the end of a given transition, transition table inserted in a rule refers to those tuples of the rule’s table that were inserted by the transition, transition table deleted refers to those tuples that were deleted, and transition tables new-updated and old-updated refer to the new and old values (respectively) of the updated tuples. A rule may refer only to transition tables corresponding to its triggering operations; note that a rule is triggered iff one or more of the corresponding transition tables is non-empty.

Rules are activated at rule assertion points. There is an assertion point at the end of each transaction, and there may be additional user-specified assertion points within a transaction (but not within SQL operations). We consider the semantics of rule processing at an arbitrary assertion point. The state change resulting from the user-generated database operations executed since the last assertion point (or start of the transaction) create the first relevant transition, and some set of rules are triggered by this transition. A triggered rule r is chosen from this set for consideration. Rule r must be chosen so that no other triggered rule has precedence over r. If r has a condition, then it is checked. If r’s condition is false, then another triggered rule is chosen for consideration. Otherwise, if r has no condition or its condition is true, then r’s action is executed. After execution of r’s action, all rules not yet considered are triggered only if their transition predicates hold with respect to the composite transition created by the initial transition and subsequent execution of r’s action. That is, these rules see r’s action as if it were executed as part of the initial transition. Rules already considered (including r) have already “processed” the initial transition; thus, they are triggered again only if their transition predicates hold with respect to the transition created by r’s action. From the new set of triggered rules, a rule r’ is chosen for consideration such that no other triggered rule has precedence over r’. Rule processing continues in this fashion.

At an arbitrary time in rule processing, a given rule is triggered if its transition predicate holds with respect to the (composite) transition since the last time it was considered. If it has not yet been considered, it is triggered if its transition predicate holds with respect to the transition since the last rule assertion point or start of the transaction. The values of transition tables in rule conditions and actions always reflect the rule’s triggering transition. When there are no triggered rules with true conditions, rule processing terminates.

3 Denotational Semantics for the Starburst Rule Language

We take as given a semantics for rule conditions and actions, which in this case is SQL predicates and operations.¹ In the informal description of Section 2, the choice of which rule to consider when multiple highest-priority rules are triggered is said to be “arbitrary”. We assume initially that this rule selection is performed by a deterministic algorithm [ACL91], and we take the semantics of

¹It may be presumptuous to assume a semantics for SQL, but we do not intend to tackle this issue here.
this algorithm as given. In Section 3.4 we modify our semantics for the case when rule selection nondeterministically chooses any eligible rule.

### 3.1 Domains

- Let $S$ be the domain of database states.

  If $s$ is a state in $S$, then $s = \{t_1, t_2, \ldots, t_n\}$ where each $t_i$ is a tuple. We assume that tuples include unique, non-reusable identifiers, and for simplicity (without loss of generality) we assume that a tuple $t_i$'s identifier also identifies $t_i$'s table.

- Let $\Delta$ be the domain of sets of database changes.

  If $\delta$ is a set of changes in $\Delta$, then $\delta = [I, D, U]$. $I = \{(tid_1, v_1), \ldots, (tid_n, v_n)\}$ where each $tid_i$ is the identifier of a (inserted) tuple and each $v_i$ is the null value. $D = \{(tid_1, v_1), \ldots, (tid_n, v_n)\}$ where each $tid_i$ is the identifier of a (deleted) tuple and each $v_i$ is a (value of the deleted) tuple with identifier $tid_i$. $U = \{(tid_1, v_1), \ldots, (tid_n, v_n)\}$ where each $tid_i$ is the identifier of a (updated) tuple and each $v_i$ is a (old value of the updated) tuple with identifier $tid_i$. No tuple identifier appears more than once in $I \cup D \cup U$.

- Let $R$ be the domain of production rules.

  If $r$ is a rule in $R$, then $r$ is a function that takes as arguments a set of changes $\delta$ and a database state $s$. It returns a boolean value, a new set of changes, and a new database state. That is:

  \[ r : \Delta \times S \rightarrow \{true, false\} \times \Delta \times S \]

  \[ r(\delta, s) \downarrow 1 = true \text{ if } r \text{ is triggered by the changes in } \delta \text{ (using the obvious definition).} \]

  If $r(\delta, s) \downarrow 1 = false$ then $r(\delta, s) \downarrow 2 = [\emptyset, \emptyset, \emptyset]$ and $r(\delta, s) \downarrow 3 = s$. If $r(\delta, s) \downarrow 1 = true$ then $r(\delta, s) \downarrow 2$ and $r(\delta, s) \downarrow 3$ are the results of executing rule $r$ starting with database state $s$ and using $\delta$ for $r$’s transition tables (with the assumed semantics of SQL). “Executing” rule $r$ includes evaluating its condition and, if true, executing its action. Note that if $r$’s condition is false then $r(\delta, s) \downarrow 2 = [\emptyset, \emptyset, \emptyset]$ and $r(\delta, s) \downarrow 3 = s$.

- Let $O$ be the domain of rule orderings.

  If $o$ is an ordering in $O$, then $o = \{r_i > r_j, r_k > r_l, \ldots\}$ where each $r_i$ is in $R$ and $>$ is transitive and irreflexive (but not necessarily total).

- Let $RC$ be the domain of sets of rule-changes pairs.

  $RC \subset P(R \times \Delta)$, where $P$ is the powerset operator. (I.e. if $A$ is a set, then $P(A)$ is the set of all subsets of $A$.) If $rc$ is a set of rule-changes pairs in $RC$, then $rc = \{(r_1, \delta_1), \ldots, (r_n, \delta_n)\}$, and no rule appears more than once in $rc$.

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2We use $i$ to denote the $i$th element in a sequence.
3.2 Supporting Functions

For each function, we give an informal description, a specification of the argument and result domains, and the function definition. Function definitions are given using the lambda-calculus \[\text{HS86}\]. Briefly, \(\lambda x_1,\ldots, x_n. E\) denotes a function that takes argument values \(v_1, \ldots, v_n\) and returns the result of evaluating expression \(E\) with all free occurrence of \(x_i\) in \(E\) replaced by \(v_i\), \(1 \leq i \leq n\). Note that \(E\) may itself be a \(\lambda\)-expression.

- Function \(\text{Run-Rule}\) takes a rule \(r_i\), a database state \(s_i\) and a set of rule-changes pairs \(rc\). It returns (1) the new database state resulting from executing rule \(r_i\) starting with database state \(s_i\) and using \(r_i\)'s changes in \(rc\) for \(r_i\)'s transition tables, and (2) the new set of rule-changes pairs that contains the net effect of \(r_i\)'s changes with each set of changes in \(rc\) (except for \(r_i\)'s, which is replaced by \([\emptyset, \emptyset, \emptyset]\)). \(\text{Run-Rule}\) is undefined on and is never applied to arguments in which \(r_i\) does not appear in \(rc\).

\[\text{Run-Rule} : \mathcal{R} \times \mathcal{S} \times \mathcal{RC} \rightarrow \mathcal{S} \times \mathcal{RC}\]

\[\text{Run-Rule} = \lambda r_i, s, \{<r_1, \delta_1>, \ldots, <r_n, \delta_n>\}. \]

\[\langle r_1(\delta_1, s) \downarrow 3, \text{Add-Changes}(r_1, r_1; \delta_1, s) \downarrow 2, \{<r_1, \delta_1>, \ldots, <r_n, \emptyset>\}, \ldots, \langle r_n, \delta_n\rangle \}\]

- Function \(\text{Add-Changes}\) takes a set of changes \(\delta\) and a set of rule-changes pairs \(rc\). It returns the modified set of rule-changes pairs that contains the net effect of \(\delta\) with each set of changes in \(rc\).

\[\text{Add-Changes} : \Delta \times \mathcal{RC} \rightarrow \mathcal{RC}\]

\[\text{Add-Changes} = \lambda \delta, \{<r_1, \delta_1>, \ldots, <r_n, \delta_n>\}. \{<r_1, \text{Net-Effect}(\delta_1, \delta)>, \ldots, <r_n, \text{Net-Effect}(\delta_n, \delta)>\}\]

- Function \(\text{Net-Effect}\) takes two sets of changes and returns the set of changes that is their net effect. (This is similarly defined in \[\text{WF90}\].)

\[\text{Net-Effect} : \Delta \times \Delta \rightarrow \Delta\]

\[\text{Net-Effect} = \lambda [I_1, D_1, U_1], [I_2, D_2, U_2].\]

\[\{((I_1 -^* D_2) \cup I_2, D_1 \cup (D_2 -^* I_1), (U_1 -^* D_2) \cup^* (U_2 -^* I_1))\}\]

where \(S_1 -^* S_2\) is defined as \(\{<\text{tid}, v> \in S_1 \mid \text{tid} \text{ does not appear in } S_2\}\) and \(S_1 \cup^* S_2\) is defined as \(S_1 \cup (S_2 -^* S_1)\).

- Function \(\text{Choose-Triggered}\) takes a set of rule-changes pairs \(rc\) and a rule ordering \(o\). It returns a rule \(r\) in \(rc\) that is triggered by its changes in \(rc\) and is such that no other rule in \(rc\) with precedence over \(r\) in \(o\) is triggered by its changes in \(rc\). \(\text{Choose-Triggered}\) is undefined on and is never applied to a set \(rc\) containing no triggered rules.

\[\text{Choose-Triggered} : \mathcal{RC} \times \mathcal{O} \rightarrow \mathcal{R}\]

\[\text{Choose-Triggered} = \lambda \{<r_1, \delta_1>, \ldots, <r_n, \delta_n>\}, o. \text{Select}(\text{Eligible}(\{<r_1, \delta_1>, \ldots, <r_n, \delta_n>\}, o))\]
• Function *Eligible* takes a set of rule-changes pairs \( \mathcal{R} \) and a rule ordering \( o \). It returns all rules \( r \) in \( \mathcal{R} \) that are triggered by their changes in \( \mathcal{R} \) and are such that no other rule in \( \mathcal{R} \) with precedence over \( r \) in \( o \) is triggered by its changes in \( \mathcal{R} \).

\[
\text{Eligible} : \mathcal{R} \times O \to P(\mathcal{R}) \quad \text{(recall that } P \text{ is the powerset operator)}
\]

\[
\text{Eligible} = \lambda \{ \langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle \}, o.
\begin{align*}
\{ r_i & \mid 1 \leq i \leq n \land r_i(\delta_i, \emptyset) \downarrow 1 = \text{true} \land \\
& \{ r_j \mid 1 \leq j \leq n \land r_j(\delta_j, \emptyset) \downarrow 1 = \text{true} \land r_j > r_i \in o \} = \emptyset \}
\end{align*}
\]

• Function *Select* takes a set of rules and deterministically chooses one.

\[
\text{Select} : P(\mathcal{R}) \to \mathcal{R}
\]

\( \text{Select} \) is undefined on and is never applied to the empty set. As mentioned above, we take as given a definition for this function. In Section 3.4 we modify the semantics for the case when this function is nondeterministic.

### 3.3 The Meaning Function

The semantics is denoted by meaning function \( M \). \( M \) takes a set of rules \( R \in P(\mathcal{R}) \) and a rule ordering \( o \in O \) such that all rules in \( o \) also are in \( R \). The meaning of \( R \) and \( o \), denoted \( M[R, o] \), is a function, call it \( \phi \). Function \( \phi \) takes a set of changes \( \delta \) and a database state \( s \). It returns the new database state that results from processing the rules in \( R \) starting with initial changes \( \delta \) and state \( s \), and using the ordering in \( o \). If rule processing does not terminate then \( \phi \) returns \( \bot \) (bottom). \( M \) is defined as follows.

\[
M : P(\mathcal{R}) \times O \to \Delta \times S \to S \cup \{ \bot \}
\]

\[
M \{ \{ r_1, r_2, \ldots, r_n \}, o \} = \lambda \delta, s. M'(o) \{ \langle s, \text{Distrib}(\delta, \{ r_1, r_2, \ldots, r_n \} \rangle) \}
\]

Function *Distrib* takes a set of changes \( \delta \) and a set of rules \( R \). It returns the set of rule-changes pairs that results from distributing \( \delta \) to each rule in \( R \).

\[
\text{Distrib} : \Delta \times P(\mathcal{R}) \to \mathcal{R}
\]

\[
\text{Distrib} = \lambda \delta, \{ r_1, r_2, \ldots, r_n \}. \{ \langle r_1, \delta \rangle, \ldots, \langle r_n, \delta \rangle \}
\]

Finally, \( M' \) takes an ordering \( o \) and returns the least fixed point of a function \( F \). \( F \) takes a database state \( s \) and a set of rule-changes pairs \( \mathcal{R} \). It returns \( s \) if no rules in \( \mathcal{R} \) are triggered by their changes in \( \mathcal{R} \). Otherwise, it calls function *Choose-Triggered* to choose a rule \( r_i \), then applies itself to the new state and set of rule-changes pairs that result from calling function *Run-Rule* with \( r_i \), \( s \), and \( \mathcal{R} \).

\[
M' : O \times S \times \mathcal{R} \to S
\]

\[
M' = \lambda o. \text{Least-Fixed-Point}(\lambda F. \\
\lambda \{ s, \{ \langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle \} \} . \\
\text{if } \text{Eligible}(\{ \langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle \}, o) = \emptyset \text{ then } s \\
\text{else let } r_i = \text{Choose-Triggered}(\{ \langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle \}, o) \in \\
F(\text{Run-Rule}(r_i, s, \{ \langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle \})))
\]

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3.4 Modified Semantics for Nondeterministic Rule Selection

When rule selection is nondeterministic, the function $\phi$ produced by meaning function $M$ returns the set of all database states (rather than a single state) that can result from processing the rules in $R$ starting with initial changes $\delta$ and state $s$, and using the ordering in $o$. If rule processing cannot terminate (regardless of which rules are selected) then $\phi$ returns the empty set. $M$ is defined as follows.

$$M : P(R) \times O \rightarrow \Delta \times S \rightarrow P(S)$$

$$M[[r_1, r_2, \ldots, r_n], o] = \lambda \delta, s. M'(o)(\langle s, Distrib(\delta, \{r_1, r_2, \ldots, r_n\})\rangle)$$

Function $Distrib$ is defined as in Section 3.3. $M'$ applied to an ordering $o$ is the least fixed point of a function $F$ that takes a database state $s$ and a set of rule-changes pairs $rc$. $F$ returns $\{s\}$ if no rules in $rc$ are triggered by their changes in $rc$. Otherwise, $F$ calls function $Eligible$ (from Section 3.2) to find all eligible triggered rules, then takes the union of the results of applying itself, for each eligible rule $r_i$, to the new state and set of rule-changes pairs that result from calling function $Run-Rule$ with $r_i$, $s$, and $rc$. Note that function $Choose-Triggered$ (and consequently function $Select$) is not used here.

$$M' : O \rightarrow S \times RC \rightarrow P(S)$$

$$M' = \lambda o. \text{Least-Fixed-Point}(\lambda F.
$$

$$\lambda \langle s, \{\langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle\}\rangle.$$

$$\text{if } Eligible(\{\langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle\}, o) = \emptyset \text{ then } \{s\}$$

$$\text{else let } R' = Eligible(\{\langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle\}, o) \text{ in }$$

$$\bigcup_{r_i \in R'} F(\text{Run-Rule}(r_i, s, \{\langle r_1, \delta_1 \rangle, \ldots, \langle r_n, \delta_n \rangle\}))$$

Note that this semantics does not distinguish between the case in which rule processing always terminates and the case in which rule processing may or may not terminate (depending on which rules are selected): In both cases, the function produced by $M$ returns a non-empty set of database states. We can extend our semantics to include this distinction. Let there be an additional domain $T$ containing one element called term: $T = \{\text{term}\}$. We modify the function $\phi$ produced by meaning function $M$ to return the set of all database states that can result from processing the rules in $R$ along with either term or $\bot$. If term is returned then rule processing always terminates; if $\bot$ is returned then rule processing may not terminate.

$$M : P(R) \times O \rightarrow \Delta \times S \rightarrow P(S) \times (T \cup \{\bot\})$$

$$M[[r_1, r_2, \ldots, r_n], o] = \lambda \delta, s. M'(o)(\langle s, Distrib(\delta, \{r_1, r_2, \ldots, r_n\})\rangle)$$

We modify the function $F$ in $M'$ to return $\{s\}$, term if no rules are triggered. Otherwise, $F$ calls function $Eligible$ to find all eligible triggered rules, then takes the "composition" (defined below) of the results of applying itself, for each eligible rule $r_i$, to the new state and set of rule-changes pairs that result from calling function $Run-Rule$ with $r_i$, $s$, and $rc$.

$$M' : O \rightarrow S \times RC \rightarrow P(S) \times (T \cup \{\bot\})$$
\[ M' = \lambda \alpha. \text{Least-Fixed-Point}(\lambda F. \lambda \langle s, \{< r_1, \delta_1 >, \ldots, < r_n, \delta_n > \} \rangle. \]
\[
\quad \text{if } \text{Eligible}(\{< r_1, \delta_1 >, \ldots, < r_n, \delta_n > \}, o) = \emptyset \text{ then } \{\{s\}, \text{term}\}
\]
\[
\quad \text{else let } R' = \text{Eligible}(\{< r_1, \delta_1 >, \ldots, < r_n, \delta_n > \}, o) \text{ in }
\]
\[
\quad \bigcup_{r_i \in R'} F(\text{Run-Rule}(r_i, \langle r_1, \delta_1 >, \ldots, < r_n, \delta_n >))
\]
\]
\[
\text{where } \langle S_1, t_1 \rangle \circ \langle S_2, t_2 \rangle \text{ is defined as } \langle S_1 \cup S_2, t \rangle, \text{ with } t = \bot \text{ if } t_1 = \bot \text{ or } t_2 = \bot \text{ and } t = \text{term} \text{ otherwise.}
\]

\section*{Acknowledgements}

A discussion with Louiza Raschid inspired me to work this out. Alex Aiken and Ed Wimmers made very insightful contributions. Bill Cody provided useful comments.

\section*{References}


